PERIOD-VI

M10CH11

TOPIC 11

Statistics, Ratio and Rates, and Percentages

A. STATISTICS

11.1 STATISTICS

Statistics is the branch of Mathematics which deals with the collection, presentation and analysis of numerical data and drawing conclusion on the basis of the same data.

In particular, statistics deals with how data is:

(*a*) collected;

(*b*) grouped together in a suitable way for graphical representation;

(*c*) interpreted mathematically

Definition of Terms

Discrete data

Values of discrete data are restricted to only certain, exact distinct numbers (but not measurements).

For example: A number of pupils can be 0, 1, 2, 3, 4... but not – 3, 2½, 4.7, 5.8... . Discrete data are usually found by counting.

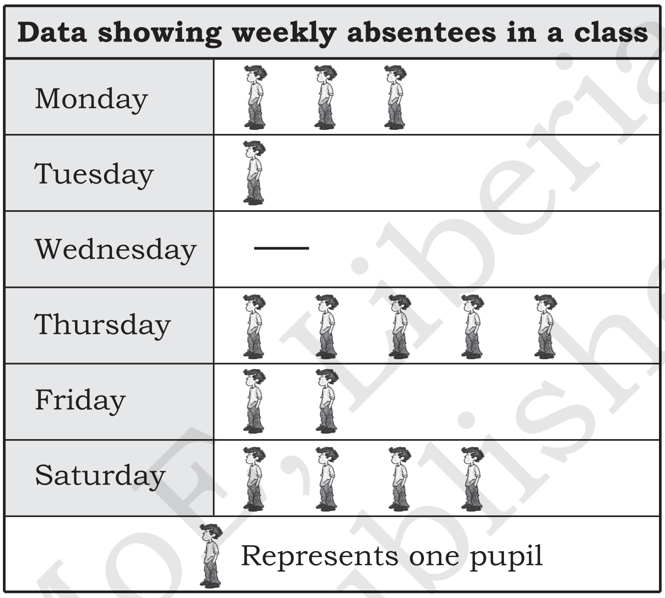
Continuous data

Values of continuous data can be any real number. Continuous data are usually found by measuring (not counting) and rounded to a suitable degree of accuracy.

For example: Weight and height can be measured to the nearest kg or cm respectively.

Collecting Data

The following table shows the data of weekly absentees in a class:



The data about the weekly absentees in a class shown earlier can tell us many things, but it cannot tell us the date which had the maximum number of absentees during a year. To find that, we need to collect the data according to the dates regarding the maximum number of absentees in each of the months during a year.

This shows that a given collection of data may not give us a specific information related to that data. For this we need to collect data keeping in mind that specific information.

Thus, *before collecting data, we need to know what we would use it for.*

ACTIVITY 1

Carrying out simple survey to collect the marks scored in an exercise out of 10

A survey was carried out in a class of 40 pupils. The collected marks are:

8	1	3	7	6	5	5	4	4	2
4	9	5	3	7	1	6	5	2	7
7	3	8	4	2	8	9	5	8	6
7	4	5	6	9	6	4	4	6	6

Note: There are two types of data—*primary data* and *secondary data.*

Primary data is the data collected directly from the source.

For example: If a person wants information about the monthly income, expenditure, number of family members, number of school going children etc. of the employees of a firm.

Secondary data is the data collected from secondary sources such as the Internet, TV, libraries, newspapers, etc.

11.2 FREQUENCY TABLES *AND* HISTOGRAMS

(*a*) Making Frequency Tables

In this section, we will learn how to make frequency tables by tallying in groups of five and write the frequencies. Let us construct frequency tables for a given data.

Example 1. *A group of 30 pupils were surveyed on which animal they would like the most to have as a pet. The results are given below:*

dog, cat, cat, fish, cat, rabbit, dog, cat, rabbit, dog, cat, dog, dog, dog, cat, rabbit, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog, rabbit, cat.

Make a frequency distribution table for this data.

Solution. For counting purposes, we use tally marks. After putting 4 tally marks vertically, we cross it as shown below and again we take the tally marks in the same manner, counting in sets of fives.

Frequency Distribution Table

Animal	Tally Marks	Number of Animals (Frequency)
Dog	~~||||~~ ~~||||~~	10
Cat	~~||||~~ ~~||||~~ |	11
Fish	|||	3
Rabbit	~~||||~~ |	6
	Total	**30**

As mentioned earlier, the frequency gives the number of times a particular entry occurs. The above table is known as *frequency distribution table* as it gives the number of times an entry occurs.

Example 2. *Consider the following marks (out of 50) obtained in Mathematics by 60 pupils of High School Grade 10.*

21, 10, 30, 22, 33, 5, 37, 12, 25, 42, 15, 39, 26, 32, 18, 27, 28, 19, 29, 35, 31, 24, 36, 18, 20, 38, 22, 44, 16, 24, 10, 27, 39, 28, 49, 29, 32, 23, 31, 21, 34, 22, 23, 36, 24, 36, 33, 47, 48, 50, 39, 20, 7, 16, 36, 45, 47, 30, 22, 17.

Make a frequency table for the above data using intervals 0–10, 10–20 and so on.

Solution. **Grouped Frequency Distribution Table**

Groups	Tally Marks	Frequency
0–10	||	2
10–20	𝍸 𝍸	10
20–30	𝍸 𝍸 𝍸 𝍸 |	21
30–40	𝍸 𝍸 𝍸 ||||	19
40–50	𝍸 ||	7
50–60	|	1
	Total	**60**

Data represented in this way is said to be *grouped* and the distribution obtained is called *grouped frequency distribution table.*

It helps us to draw meaningful inferences like:

(*i*) Most of the pupils have scored between 20 and 40.

(*ii*) Eight pupils have scored more than 40 marks out of 50 and so on.

Each of the groups: 0–10, 10–20, 20–30 etc., is called a *Class Interval* (or briefly a class).

Observe that 10 occurs in both classes, *i.e.*, 0–10 as well as 10–20. Similarly, 20 occurs in classes 10–20 and 20–30. But it is not possible that an observation (say 10 or 20) can belong simultaneously to two classes. To avoid this, we adopt the convention that the common observation will belong to the higher class, *i.e.,* 10 belongs to the class interval 10–20 (and not to 0–10).

Similarly, 20 belongs to 20–30 (and not to 10–20). In the class interval, 10–20, 10 is called the *lower class limit* and 20 is called the *upper class limit.*

Similarly, in the class interval 20–30, 20 is the *lower class limit* and 30 is the *upper class limit.*

Observe that the difference between the upper class limit and lower class limit for each of the class intervals 0–10, 10–20, 20–30 etc., is equal, (10 in this case). This difference between the upper class limit and lower class limit is called the *width* or *size* of the class interval.

(*b*) Histograms

A histogram is a graphical representation of a frequency distribution in the form of adjacent rectangles with class intervals as bases and heights proportional to frequency.

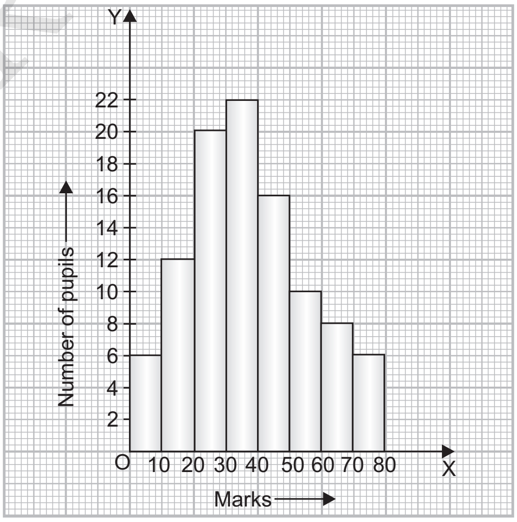
In a histogram, the width of a rectangle is significant and represents the class size.

Example 3. *The following table gives the marks scored by 100 pupils in an entrance examination.*

Marks	*0 – 10*	*10 – 20*	*20 – 30*	*30 – 40*	*40 – 50*	*50 – 60*	*60 – 70*	*70 – 80*
Number of pupils	*6*	*12*	*20*	*22*	*16*	*10*	*8*	*6*

Draw a histogram for the above data.

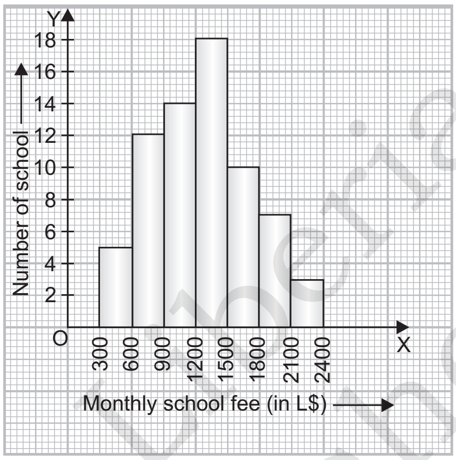
Solution. Represent the class limits along *x*-axis on a suitable scale and the frequencies (*i.e.*, number of pupils) along *y*-axis on a suitable scale. With class intervals as bases and the corresponding frequencies as heights, construct rectangles to obtain the desired histogram.



Example 4. *Construct a histogram for the following data:*

Monthly school fee (in L$)	*300-600*	*600-900*	*900-1200*	*1200-1500*	*1500-1800*	*1800-2100*	*2100-2400*
Number of school	*5*	*12*	*14*	*18*	*10*	*7*	*3*

Solution. Represent the class limits along x-axis on a suitable scale and the frequencies (i.e., number of school) along y-axis on a suitable scale. With class intervals as bases and the corresponding frequencies as heights, construct rectangles to obtain the desired histogram (see the figure).



EXERCISE 11.1

1. Carry out simple surveys to collect data for the following:

(*a*) (*i*) Number of pupils below the age of five in the families around you.

(*ii*) Performance of Liberia in football or in athletics

(*iii*) Female literacy rate in a given area

(*iv*) Highest maximum temperature of a city during a year.

(*b*) What kind of data would you need in the above situations? Unless and until you collect appropriate data, you cannot know the desired information.

(*c*) What is the appropriate data for each?

(*d*) Discuss with your friends and identify the data you would need for each. Some of this data is easy to collect and some difficult.

2. Construct frequency tables of the data collected by your classmates, *i.e.*, examination results, rainfall in a month, import and export etc.

3. Construct a frequency distribution table for the data on weights (in kg) of 20 pupils of a class using intervals 30–35, 35–40, and so on.

40, 38, 33, 48, 60, 53, 31, 46, 34, 36, 49, 41, 55, 49, 65, 42, 44, 47, 38, 39.

4. Represent the following data in the form of a histogram:

Class interval	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	4	10	16	20	15	10	5

5. Draw a histogram to represent the following data:

Class interval	10–15	15–20	20–25	25–30	30–35	35–40
Frequency	30	80	75	55	35	50

11.3 MEASURES OF CENTRAL TENDENCY (MODE, MEDIAN *AND* MEAN)

Different forms of data need different forms of representative or central value to describe it. One of these representative value is '*Mode*'. You will learn about the other representative values '*Median*' and '*Mean*' later on in this section.

Mode

The mode of a set of observations is the observation that occurs most often.

Example 5. *Find the mode of the given set of numbers:*

1, 1, 2, 4, 3, 2, 1, 2, 2, 4.

Solution. On arranging the numbers with same values together, we get

1, 1, 1, 2, 2, 2, 2, 3, 4, 4

Mode of this data is 2 because it occurs the most frequently than other observations.

Example 6. *Following are the margins of victory in the football matches of a league:*

1, 3, 2, 5, 1, 4, 6, 2, 5, 2, 2, 2, 4, 1, 2, 3, 1, 1, 2, 3, 2, 6, 4, 3, 2, 1, 1, 4, 2, 1, 5, 3, 3, 2, 3, 2, 4, 2, 1, 2.

Find the mode of this data.

Solution. Let us put the data in a tabular form:

Margins of Victory	***Tally Bars***	***Number of Matches (Frequency)***
1	𝍸 ||||	9
2	𝍸 𝍸 ||||	14
3	𝍸 ||	7
4	𝍸	5
5	|||	3
6	||	2
	Total	**40**

Looking at the table, we can quickly say that 2 is the '*mode*' since 2 has occurred the highest number of times.

Thus, the most of the matches have been won with a victory margin of 2 goals.

Note: Mode is always one of the given observations.

Median

ACTIVITY 2

Consider a group of 7 pupils with the following heights (in cm):

148, 135, 150, 140, 140, 142, 145

- The games teacher wants to divide the class into two groups.
- Each group has equal number of pupils.
- One group has pupils with height lesser than a particular height and the other group has pupils with heights greater than the particular height.

  How would she do that?

Thus, in a given data, arranged in ascending or descending order, the *median* gives us the middle observation. (if the number of observations is odd).

Example 7. *Find the median of the numbers:*

12, 16, 8, 14, 25, 11, 10, 6, 15.

Solution. We first arrange the data in ascending order as follows:

6, 8, 10, 11, **12**, 14, 15, 16, 25

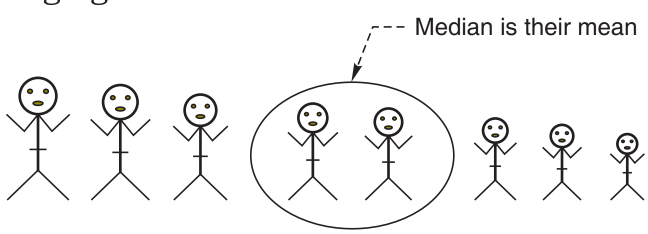
We see that the middle number of these is 12, *i.e.*, four numbers are less than 12 and four numbers are greater than 12.

Hence, the median is 12.

Note: In general, we may not get same value for median and mode.

If there is an even number of observations, the median is found by finding the mean (average) of the two middle values after they have been arranged in ascending or descending order.

The following figure illustrates this.



Observe the following set of numbers:

57, 50, 53, 62, 40, 51, 49, 61

Here, the number of observation is 8, which is even.

The above data can be arranged in descending order as below:

62, 61, 57, **53**, **51**, 50, 49, 40

Here, the two middle values are **53** and **51**.

Thus, median = $\frac{53+51}{2} = \frac{104}{2} = 52$.

Mean

The most common representative value of a group of data is the *arithmetic mean* or the *mean*. To understand this in a better way, let us look at the following Activity:

ACTIVITY 3

Consider two vessels contain 20 litres and 60 litres of milk respectively. What is the amount that each vessel would have, if both share the milk equally?

When we ask this question, we are seeking the arithmetic mean.

In the above case, the average or the arithmetic mean would be

$$\frac{\text{Total quantity of milk}}{\text{Number of vessels}} = \frac{20+60}{2} \text{ litres} = 40 \text{ litres.}$$

Thus, each vessel would have 40 litres of milk.

Thus, the average or Arithmetic Mean (A.M.) or simply mean is defined as follows:

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Example 8. *David studies for 4 hours, 5 hours and 3 hours respectively on three consecutive days. How many hours does he study daily on an average?*

Solution. The average study time of David would be

$$\frac{\text{Total number of study hours}}{\text{Number of days for which he studied}}$$

$$= \frac{4+5+3}{3} \text{ hours} = \frac{12}{3} \text{ hours} = 4 \text{ hours per day.}$$

Thus, we can say that David studies for 4 hours daily on an average.

Note: Mode, Median and Mean of a data may be three different values but always lie between the lowest and the highest observations.

EXERCISE 11.2

1. Find the mode of the following data:
   31, 35, 36, 33, 32, 35, 36, 31, 34, 33, 36, 34, 32, 32, 36, 34.
2. Heights (in cm) of 25 children are:
   168, 165, 163, 160, 163, 161, 162, 164, 163, 162, 164, 163, 160, 163, 160, 165, 163, 162, 163, 164, 163, 160, 165, 163, 162.
   What is the mode of their heights?
3. Your friend found the median and the mode of a given data. Describe and correct your friends error if any:
   35, 32, 35, 42, 38, 32, 34
   Median = 42, Mode = 32
4. Find the median of the following data:
   5, 9, 6, 4, 8, 2, 7
5. Find the median of the following data:
   42, 73, 35, 92, 67, 85, 71, 81, 51, 56
6. Find the mean of your study hours or sleeping hours for the whole week.
7. Find the mean attendance of pupils in your school for the previous year.

11.4 STEM *AND* LEAF PLOT

A plot where each data value is split into a "leaf" (usually the last digit) and a "stem" (the other digits) is called a stem and leaf plot.

For example: Observe the following stem and leaf plot:

12, 13, 24, 25, 25, 28, 28, 30, **37**, 49

Stem	**Leaf**				
1	2	3			
2	4	5	5	8	8
3	0	**7**			
4	9				

Here, we have shown how "37" would be split into "3" (stem) and "7" (leaf).

The "*stem*" values are listed down, and the "*leaf*" values are listed next to them.

This way the "*stem*" groups the scores and each "*leaf*" indicates a score within that group.

Example 9. *The heights of 28 pupils, measured to the nearest centimetres, have been found to be as follows:*

161	*150*	*154*	*165*	*168*	*161*	*154*
162	*150*	*151*	*162*	*164*	*171*	*165*
158	*154*	*156*	*172*	*160*	*170*	*153*
159	*161*	*170*	*162*	*165*	*166*	*168*

Represent it using a stem and leaf plot.

Solution. The stem and leaf plot is:

Stem	**Leaf**
15	0 4 4 0 1 8 4 6 3 9
16	1 5 8 1 2 2 4 5 0 1 2 5 6 8
17	1 2 0 0

11.5 GRAPHICAL DISPLAYS

Graphical display is a way of analysing numerical data. It exhibits the relation between data, ideas, information and concepts in a diagram. It is one of the most important learning strategies. It always depends on the type of information in a particular domain. There are different types of graphical displays. Some of them are as follows:

- Line Graphs
- Histograms
- Frequency Table
- Stem and Leaf Plot
- Bar Graphs
- Line Plot
- Circle Graph
- Box and Whisker Plot

Bar Graphs and Double Bar Graphs

Bar graph is used to display the category of data and it compares the data using solid bars to reparesent the quantities.

Example 10. *Maruti Suzuki company has following data of sales recorded for last 3 days.*

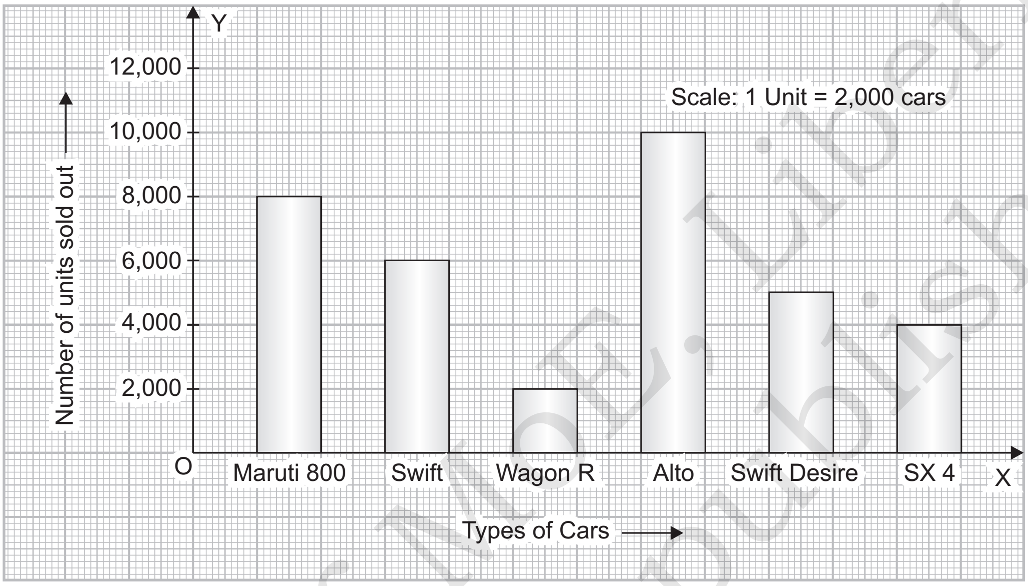
Maruti 800	***Swift***	***Wagon R***	***Alto***	***Swift Desire***	***SX 4***
8,000	*6,000*	*2,000*	*10,000*	*5,000*	*4,000*

Draw a bar graph and answer the following questions:

(*a*) *Which model recorded maximum sales?*

(*b*) *Which model recorded minimum sales?*

Solution. *Bar graph:*



(*a*) Alto model recorded maximum sales.

(*b*) Wagon R model recorded minimum sale.

Example 11. *Following table shows the number of HIV positive data of Liberia (male and female) in the particular years from January 2014 to December 2018.*

Years	*2014*	*2015*	*2016*	*2017*	*2018*
Male	*204*	*224*	*372*	*270*	*339*
Female	*474*	*412*	*685*	*516*	*570*

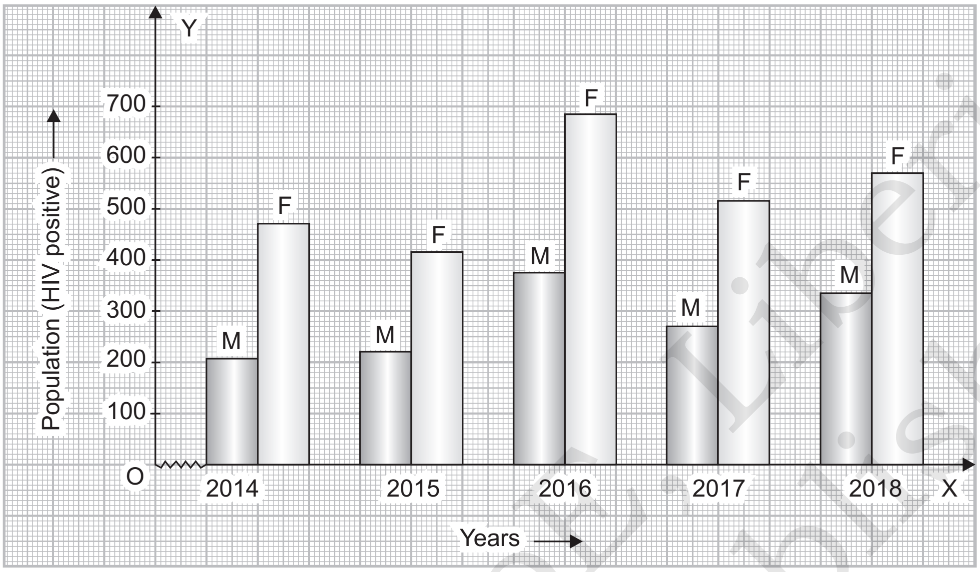
Draw a double bar graph using the above data.

(*a*) *In which year you record the maximum number of patients?*

(*b*) *In which year you record the minimum number of patients?*

Solution. We draw double bars, one bar to show number of HIV +ve cases of male and the other bar to show HIV +ve cases of female year wise.

HIV Positive cases in Liberia



(*a*) In 2016, we record the maximum number of patients.

(*b*) In 2015, we record the minimum number of patients.

Pie Chart

A pie chart shows how something is divided. A pie chart is a circular chart divided into a number of sectors. The circle represents the whole and each sector represents the various observations.

The total angle at the centre of a circle is 360°. The central angle of the sectors will be a fraction of 360°.

Example 12. *The following table shows the sales of some vehicles in a month.*

Name of Vehicle	*Bicycles*	*Scooter*	*Motorcycle*	*Car*	*Bus*	*Total*
Sale in Number	*80*	*40*	*50*	*60*	*10*	*240*

Draw a pie chart to represent the data.

Solution. Since the angle about the centre of the circle is 360°, calculate the corresponding central angles. Here the total sale = 240.

Name of Vehicle	*Sales in number*	*In Fraction*	*Central angle*
Bicycle	80	$\frac{80}{240}$	$\frac{80}{240} \times 360° = 120°$
Scooter	40	$\frac{40}{240}$	$\frac{40}{240} \times 360° = 60°$
Motorcycle	50	$\frac{50}{240}$	$\frac{50}{240} \times 360° = 75°$
Car	60	$\frac{60}{240}$	$\frac{60}{240} \times 360° = 90°$
Bus	10	$\frac{10}{240}$	$\frac{10}{240} \times 360° = 15°$
Total	**240**		**360°**

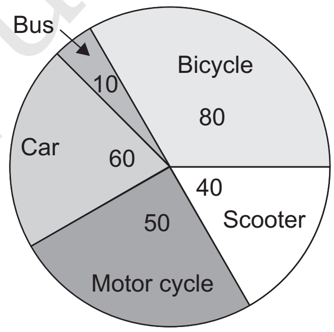
Steps of construction:

(*i*) Draw a circle of any radius.

(*ii*) Draw a horizontal radius of the circle.

(*iii*) Start with horizontal radius to form sectors with central angles of 120°, 60°, 75°, 90° and 15° respectively.

(*iv*) Mention the corresponding vehicles in the sectors.



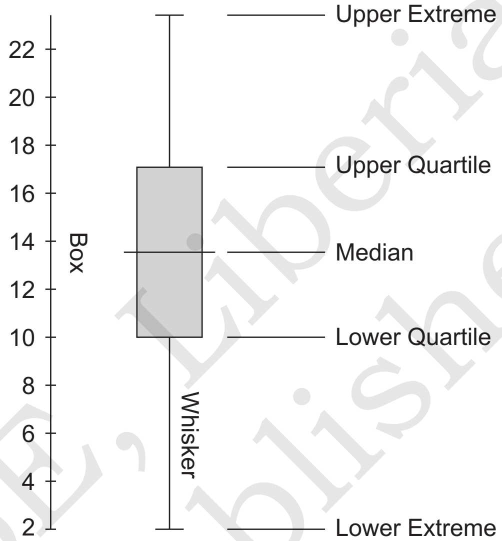
We obtain the required pie chart as shown in the figure.

11.6 BOX *AND* WHISKER PLOT

A box and whisker plot (also known as a *box plot*) is a graph that represents visually data from a five-number summary. These numbers are median, upper and lower quartile, minimum and maximum data value (extremes).

Look at the following example of box and whisker plot:

So, there are a couple of things, you should know in order to work with box plots:



- **Lower Extreme.** The smallest value in a given data set.
- **Upper Extreme.** The highest value in a given data set.
- **Median value.** The middle number in the set.
- **Lower Quartile.** Below that value, the lower 25% of the data are contained.
- **Upper Quartile.** Above that value, the upper 25% of the data are contained.
- **"Whiskers".** The lines that extend from the boxes. They are used to indicate variability out of the upper and lower quartiles.

Example 13. *Draw a simple box and whisker plot to display the data of math test results for a class of 15 pupils. Here are the results:*

91 95 54 69 80 85 88 73 71 70 66 90 86 84 73.

Solution.

Step 1: Order the data points from least to the greatest.

54 66 69 70 71 73 73 80 84 85 86 88 90 91 95

Step 2: Find the median of the data:

54 66 69 70 71 73 73 (80) 84 85 86 88 90 91 95

↑ the median

This is an odd set of data–you have 15 data points. It means the middle point is 80 as there are 7 data points above it and 7 data points below.

Step 3: Find the middle points of the two halves divided by the median (find the upper and lower quartiles).

54 66 69 (70) 71 73 73 (80) 84 85 86 (88) 90 91 95

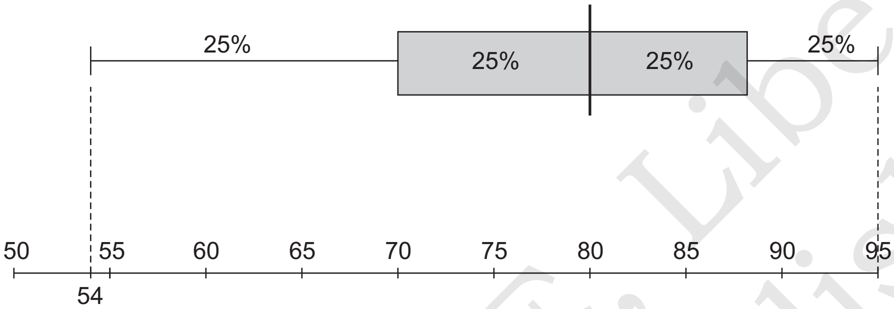
lower quartile (70) upper quartile (88)

Step 4: Find the extreme values.

Extreme value = 54 and 95.

Here the five-number summary for the class of pupils is 54, 70, 80, 88 and 95.

Now draw box-and-whisker plot.



The plot is divided into four groups: a lower whisker, a lower box half, an upper box half, and an upper whisker. Each of those groups shows 25% of the data because we have an equal amount of data in each group.

Interpreting the box and whisker plot results:

The box and whisker plot shows that 50% of the pupils have scores between 70 and 88 points.

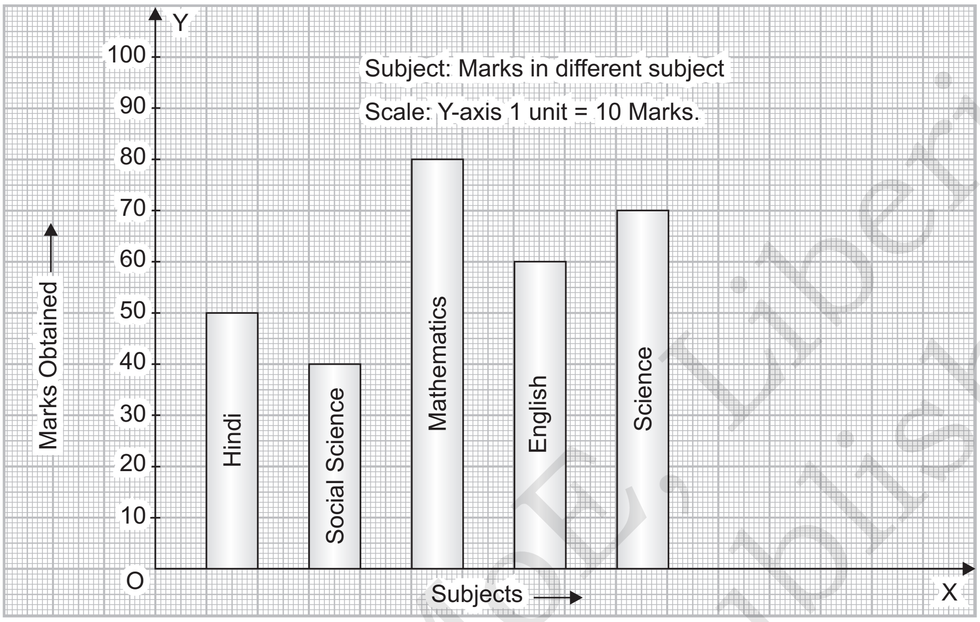
In addition, 75% scored lower than 88 points, and 50% have test results above 80 points.

EXERCISE 11.3

1. Speed of typists, in word per minute are:
76, 75, 80, 78, 75, 80, 80, 75, 76, 76, 79, 76, 77, 76, 79, 80, 79, 78, 77, 79
Draw a stem and leaf plot for this data.

2. Attendances of pupils during a month in a school are:
265, 298, 275, 280, 298, 275, 295, 295, 275, 295, 275, 298, 280, 298, 298, 275, 275, 298, 280, 265, 265, 280
Draw a stem and leaf plot for this data.

3. Read the bar graph given below and answer the following questions.



(*a*) What information does the bar graph give?

(*b*) In which subject the pupil scored maximum marks?

(*c*) In which subject the pupil scored the least marks?

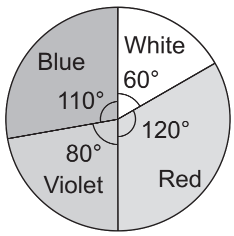
(*d*) In which subject the pupil scored more than English but less than Mathematics?

4. Following table shows population of male and female in some of the cities in a particular year.

Cities population in thousands	Coldwell	Buutuo	Bopolu	Arthington
Male (in thousands)	860	415	315	290
Female (in thousands)	810	390	290	285

Draw a double bar graph.

5. Samuel has 36 coloured marbles shown in the given pie chart. Study it carefully and answer the following questions.

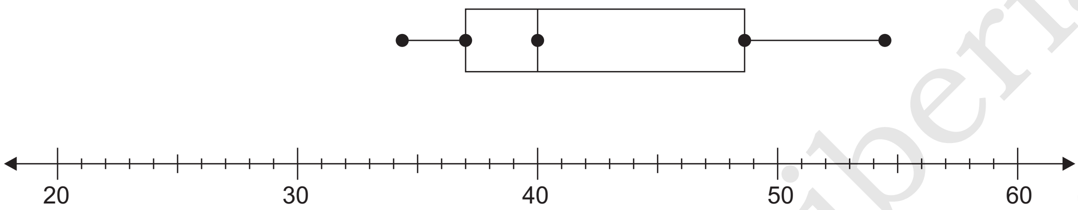


(*a*) Which colour marbles are the most?

(*b*) What is the ratio of white to red marbles Samuel has?

(*c*) How many violet marbles are there?

6. An electronic gadgets distributor distributes various brands of mobiles to retailers. The data for the number of smartphones distributed in nine months (Jan-Sep) are collected to make a box-and-whisker plot. Read the plot and answer the questions.



(*a*) Write the median form the above given plot.
(*b*) What is the least number of smartphones distributed?
(*c*) Write the third quartile from the given plot.

7. Draw the box and whisker plot to display the data: 3, 7, 8, 5, 12, 14, 21, 13, 18.

11.7 AVERAGE

The average of a list of data is the expression of the central value of a set of data. Mathematically, it is defined as the mean value which is equal to the sum of the number of a given set of values divided by the total number of values present in the set. In statistics, the average of a given set of numerical data is known as mean.

For example: The average of 3, 4 and 5 is $\frac{3+4+5}{3} = \frac{12}{3} = 4$

Here, 4 is the central value of 3, 4 and 5.

Thus, the meaning of average is to find the mean value of a group of numbers.

$$\text{Average} = \frac{\text{Sum of Values}}{\text{Number of Values}}$$

Consider, we have n number of values such as $x_1, x_2, x_3, \ldots, x_n$.

Then, average of the given data = $\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$.

Example 14. *If there are a group of numbers that is 20, 21, 23, 22, 21, 20, 23, then find the average of these numbers.*

Solution. Since, average = $\frac{\text{Sum of Values}}{\text{Number of Values}}$

$$\therefore \quad \text{Average} = \frac{20+21+23+22+21+20+23}{7} = \frac{150}{7} = 21.42$$

Example 15. *Find the average of 3, –7, 6, 12, –2.*

Solution. Sum of the numbers $= 3 + (-7) + 6 + 12 + (-2)$

$= 3 - 7 + 6 + 12 - 2$

$= 21 - 9 = 12$

Number of numbers = 5

$\therefore$ Average $= \frac{12}{5} = 2.4.$

EXERCISE 11.4

1. What is average? Give an example.
2. What is the average formula?
3. Is the average and mean of numbers same?
4. What is the average of 10, 15, 20, 100?
5. Find the average of first five prime numbers.
6. If the heights of males in a group are: 5.5, 5.3, 5.7, 5.9, 6, 5.10, 5.8, 5.6, 5.4, 6. Then find the average height.

B. RATIO *AND* RATES

11.8 RATIO

A ratio is a comparison of two or more similar quantities. It expresses what part of one is contained in the other. It has no unit. If a and b are two similar quantities. Then a/b or $a : b$ (read as a is to b) is called their ratio. The quantities a and b are known as terms of the ratio. The first term is known as *antecedent* and the second term is known as consequent.

Note: A ratio is not changed by multiplying or dividing the antecedent and the consequent by the same non zero number.

Example 16. *Find the value of x for which*

(*a*) $x + 7 : x + 4 = 3 : 2$ (*b*) $5x + 15 : 2x + 3 = 10 : 3$

Solution. (*a*) Given $\frac{x+7}{x+4} = \frac{3}{2}$

$\Rightarrow \quad 2(x + 7) = 3(x + 4) \quad \Rightarrow \quad 2x + 14 = 3x + 12$

$\Rightarrow \quad 14 - 12 = 3x - 2x \quad \Rightarrow \quad x = 2$

(*b*) Given $\frac{5x+15}{2x+3} = \frac{10}{3}$

$\Rightarrow \quad 3(5x + 15) = 10(2x + 3) \quad \Rightarrow \quad 15x + 45 = 20x + 30$

$\Rightarrow \quad 45 - 30 = 20x - 15x \quad \Rightarrow \quad 15 = 5x$

$\Rightarrow \quad x = 3$

Example 17. *Express each ratio as a fraction in its lowest form.*

(*a*) *25 cm : 3 m* (*b*) *48 s : 5 minutes*

Solution. (*a*) Using 1 m = 100 cm, we have 3 m = 300 cm. Therefore, the given ratio becomes 25 cm : 300 cm or $\frac{25}{300}$ or $\frac{1}{12}$.

(*b*) Using 1 minute = 60 s, we have 5 minutes = 300 s. Therefore, the given ratio becomes 48 s : 300 s or $\frac{48}{300}$ or $\frac{12}{75}$ or $\frac{4}{25}$.

11.9 SCALE DRAWING

You know that a map is a reduced represntation of a very large region. A scale is usually given at the top or bottom of every map. The scale shows a relationship between actual length and the length represented on the map. The scale of a map is thus the ratio of the distance between two points on the map to the actual distance between those two points on land (the ground). The ratio is often in the form 1 : *n*.

Note: A *scale* is used to reduced the actual distance between two points on a map.

If the scale of a map is 1 : *n*, then;

1. The *distance on the ground or real length*

   = *n* × distance on the map.

2. The *distance on the map*

   $= \frac{\text{distance on the ground or real length}}{n}$

Example 18. *The height of a tower of a church building in a scale drawing is 2 cm. If the scale is 1 cm to 20 m. How tall is the actual tower?*

Solution. Let actual height of the tower = h cm.

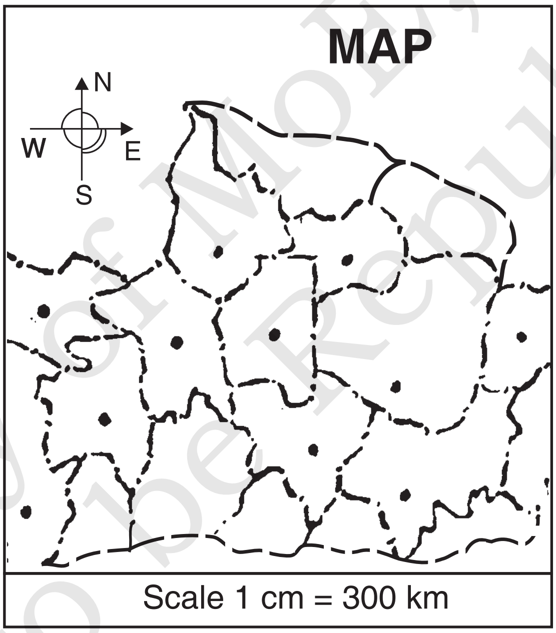
$\therefore$ 20 m = 2000 cm [$\because$ 1 m = 100 cm]

then 1 : 2000 = 2 : h *i.e.,* $\frac{1}{2000} = \frac{2}{h}$

$\Rightarrow$ $h = 2 \times 2000$ $\Rightarrow$ $h = 4000$

So, actual height of the tower = 4000 cm = $\frac{4000}{100}$ = 40 m.

Example 19. *The scale of a map is given as 1 : 30000000. Two cities are 4 cm apart on the map. Find the actual distance between them.*



Solution. Let the map distance be x cm and actual distance be y cm, then

$$1 : 30000000 = x : y \Rightarrow \frac{1}{3\times10^7} = \frac{x}{y}$$

Since $x = 4$ so, $\frac{1}{3\times10^7} = \frac{4}{y}$

$\Rightarrow$ $y = 4 \times 3 \times 10^7 = 12 \times 10^7$ cm = 1200 km.

Thus, two cities, which are 4 cm apart on the map, are actually 1200 km away from each other.

EXERCISE 11.5

1. Find the ratio between 10 kg and 15 kg.
2. Find which is greater
   (*a*) 2 : 3 or 3 : 4 (*b*) 5 : 7 or 7 : 9 (*c*) 1 : 4 or 2 : 9
3. Find the value of x for which $5x + 1 : 2x + 3 = 1 : 2$.
4. Two numbers are in the ratio 1 : 2. When 4 is added to each, the ratio becomes 2 : 3. Find the numbers.
5. Monthly incomes of two persons are in the ratio of 4 : 5. Their monthly expenditures are in the ratio of 7 : 9. If each saves L$ 50 a month, find their monthly incomes.
6. A map is drawn to a scale of 1 : 20000.
   (*a*) Find the distance on the map between two points that are 12 km apart.
   (*b*) The distance on the map between two villages is 25 cm. Find the real distance between the two villages.
7. A map of a large town is drawn to the scale of 1 : 100000. What is the distance in kilometres (km) represented by a line segment 4 cm on the ground?
8. The length of a field 1.2 km long is represented on a map by a line 40 mm long. What is the scale of the map?

11.10 RATES

A rate is a ratio that is used for comparing two different kinds of quantities which have different units.

For example: If there is a comparison of books to pupils. These are comparing two different units (an object and a person). If the rate is 4 : 1, this indicates that there are 4 books for 1 pupil.

Also, if William drives 300 miles in 3 hours. Then this is a rate. It can be converted to a *unit rate* by dividing by the second value (the denominator) when each number is divided by 3, a unit rate of 100 miles for every 1 hour or 100 miles per hour is calculated,

i.e., $$\frac{300}{3} : \frac{3}{3} = \frac{100}{1}$$

In fact, unit rate is said to be the amount of something in each unit or per unit.

Note: **1.** A rate is a special kind of ratio that is used to show the comparison of two different units of measurement while a ratio compares the same units.

2. A rate is a ratio because it compares two numbers, yet a ratio cannot be a rate because it only compares the same units.

Example 20. *A man travels 20 kilometers in 2 hours, find the unit rate.*

Solution. In 2 hours a man travels 20 kilometers

In 1 hour the man will travel = $\frac{20}{2}$ = 10 kilometer

$\therefore$ The required unit rate = 10 km/hour.

Example 21. *If a car uses 2 hours to consume 10 litres of petrol, find its unit rate and then find in*

(*a*) *5 hours* (*b*) *30 minutes*

Solution. In 2 hours the car consume petrol = 10 litres

In 1 hour the car will consume petrol = $\frac{10}{2}$ = 5 litres

$\therefore$ Unit rate = 5 litres per hour

(*a*) In 5 hours the car will consume petrol = 5 × 5 = 25 litres

(*b*) In 30 minutes = $\frac{30}{60}$ hour = $\frac{1}{2}$ hour

The car will consume petrol = $\frac{1}{2} \times 5$ = 2.5 litres.

11.11 TRAVEL GRAPHS USING RATES

Travel Graphs are the line graphs which are used to describe the motion of objects (such as bicycle, car, train, bike etc).

1. A *distance-time graph* shows the relationship between the distance (from a fixed point) and the time of a journey. The distance travelled is represented on the vertical axis and the time taken to travel the distance is represented on the horizontal axis.

2. A *speed-time graph* shows the relationship between the speed and the time of a Journey. The speed is represented on the vertical axis and the time taken to travel on the horizontal axis.

Reading a travel graph. In distance-time graph, the speed of an object at any instant is given by

$$\text{Speed} = \frac{\text{Distance travalled}}{\text{Time taken}};$$

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Gradient: The gradient of a straight line joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

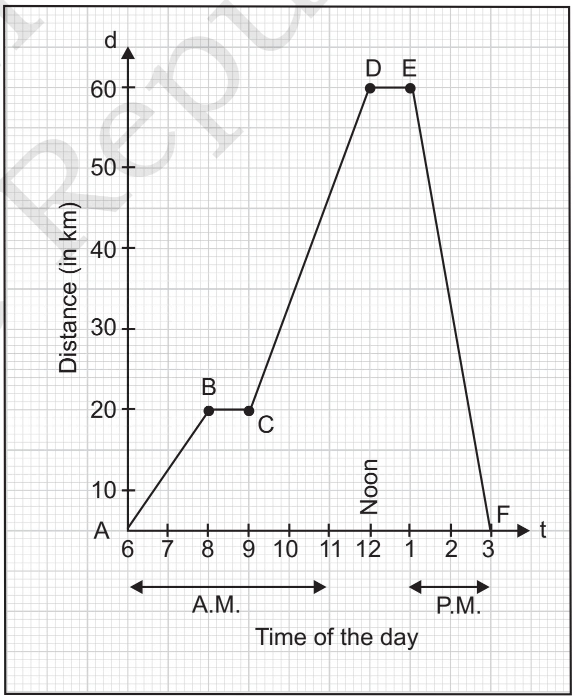
$$\text{Gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

The *gradient* of a distance-time graph represents the *speed*.

The gradient of a speed-time graph represents the *acceleration*.

Note: The gradient of a horizontal line is always zero.

Example 22. *Consider the distance time graph of a cyclist as shown in the figure.*



Answer the following questions:

(*a*) *When the cyclist leave home?*

(*b*) *When did the cyclist return home?*

(*c*) *How far away from home was he at 9 a.m.?*

(*d*) *How far away from home was he at noon?*

(*e*) *How far away from was he at 1 p.m.?*

(*f*) *At what times did he take a rest?*

(*g*) *How far away from home was he at 2 p.m.?*

(*h*) *How far away from home was he at 3 p.m.?*

(*i*) *How far away from home was he at 8 a.m.?*

(*j*) *Find his speed from:*

(*i*) *6 a.m. to 8 a.m.*

(*ii*) *12 noon to 1 p.m.*

(*iii*) *1 p.m. to 3 p.m.*

(*iv*) *When was the cyclist travelling most quickly?*

Solution. Looking at the graph,

(*a*) The cyclist left home at 6 a.m.

(*b*) The cyclist returned home at 3 p.m.

(*c*) At 9 a.m., he was 20 km away from his home.

(*d*) At noon (i.e., at 12), he was 60 km away from his home.

(*e*) At 1 p.m., he was again 60 km away from his home.

(*f*) He took rest between noon and 1 p.m.

(*g*) At 2 p.m., he was 35 km away from his home.

(*h*) At 3 p.m., he reached his home, so he was zero km away from his home.

(*i*) At 8 a.m. he was 20 km away from his home.

(*j*) (*i*) Let A(6 a.m., 0 km) and B(8 a.m., 20 km) represent the coordinates of A and B respectively.

∴ Required speed = gradient of the straight line AB

$$= \frac{20\text{ km} - 0\text{ km}}{8\text{ a.m.} - 6\text{ a.m.}} = \frac{20}{2}\text{ km/hr}$$

$$= 10\text{ km/hr}$$

(*ii*) Let D(12 (noon), 60 km) and E (1 p.m.; 60 km) represent the coordinates of D and E respectively.

∴ Required speed = gradient of the straight line DE

$$= \frac{60\text{ km} - 60\text{ km}}{1\text{ p.m.} - 12\text{ (noon)}} = 0\text{ km/hr}$$

(*iii*) Let the coordinates of E and F are E(1 p.m., 60 km) and F(3 p.m., 0 km) respectively.

∴ Required speed = gradient of the straight line EF

$$= \frac{0\text{ km} - 60\text{ km}}{3\text{ p.m.} - 1\text{ p.m.}} = -\frac{60}{2}\text{ km/hr}$$

$$= -30\text{ km/hr}$$

Here negative sign indicates the motion in opposite direction. Thus, the cyclist is moving towards home at a speed of 30 km/hr.

(*iv*) Using part *j*(*i*), *j*(*ii*) and *j*(*iii*), we observe that the cyclist was travelling the most quickly between 1 p.m. and 3 p.m.

11.12 CONVERSION GRAPHS USING RATES

Conversion graphs are straight line graphs that show a relationship between two units. It can be used to convert from one to another.

In order to draw a conversion graph:

1. Draw axes and choose which units should be represented by the horizontal and vertical.

2. Use information provided to plot points that represent the conversion between the units.

3. Draw a straight line through the points ensuring it is long enough to span the width and height of the axes.

Example 23. *Daniel's taxi calculate the cost of a journey using the following conversion graph.*

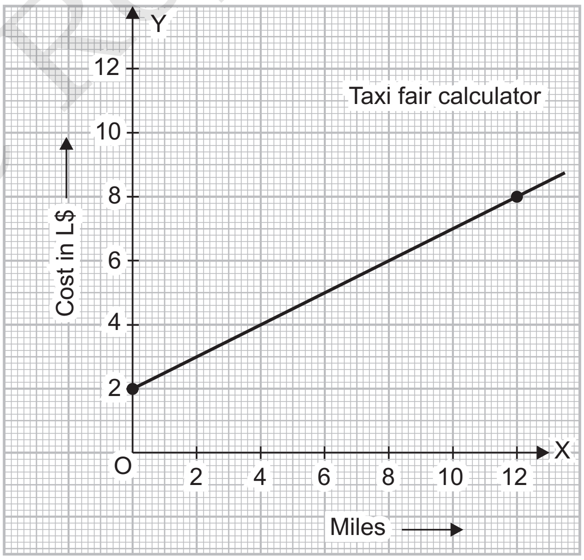
Read the graph and answer the following questions:

(*i*) *What is the fix rate of the journey?*

(*ii*) *If a journey cost L$ 10, how many miles would you expect to have travelled? You should show your answer on the graph.*

(*iii*) *Calculate the cost of a 30 mile journey.*

(*iv*) *Calculate the distance travelled when the journey costs L$ 15.*



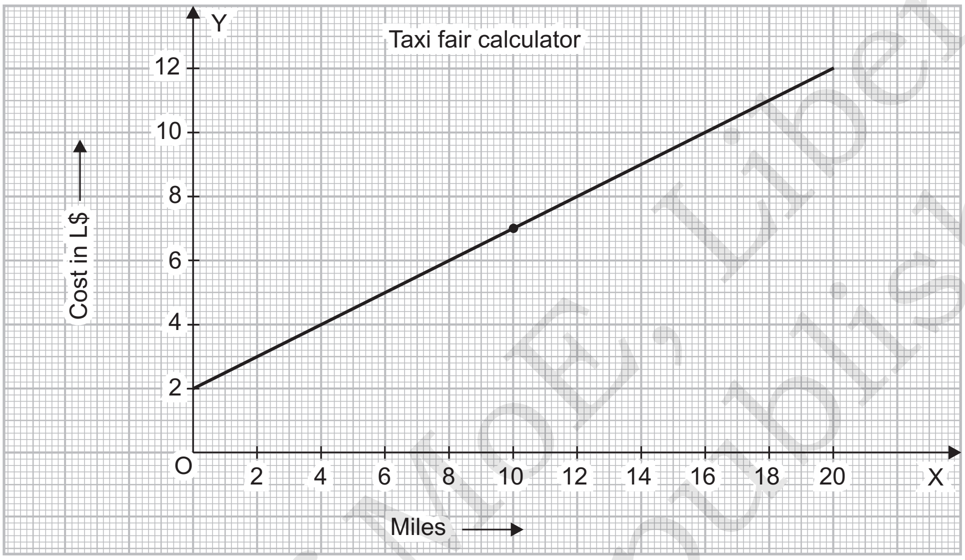
Solution. (*i*) From the graph, 0 mile = L$ 2.

This is a flat rate regardless of the distance travelled

∴ The fix rate is L$ 2.

(*ii*) As the graph doesn't pass through (0, 0) to undertake a conversion that is outside the scale on the graph a different approach must be taken.

It may be possible to extend the graph further.



We can observe from this extanded graph that L$10 would be the charge for a journey that is 16 miles long.

Note: If it is not possible to extend the graph, we will need to understand some calculations.

(*iii*) We know that a 0 miles journey costs L$ 2

We know that a 10 miles journy costs L$ 7

So each 10 miles travelled costs L$ 7 – L$ 2 = L$ 5

30 miles = 10 miles × 3, so that cost of 30 miles = L$ 5 × 3 = L$ 15

Add on the flat charge of L$ 2 for each journey:

L$ 15 + L$ 2 = L$ 17.

(*iv*) From the graph we can see that L$ 2 = 0 mile and L$ 3 = 2 miles.

This means that after the flat rate of L$ 2, L$ 1 is added on for every 2 miles.

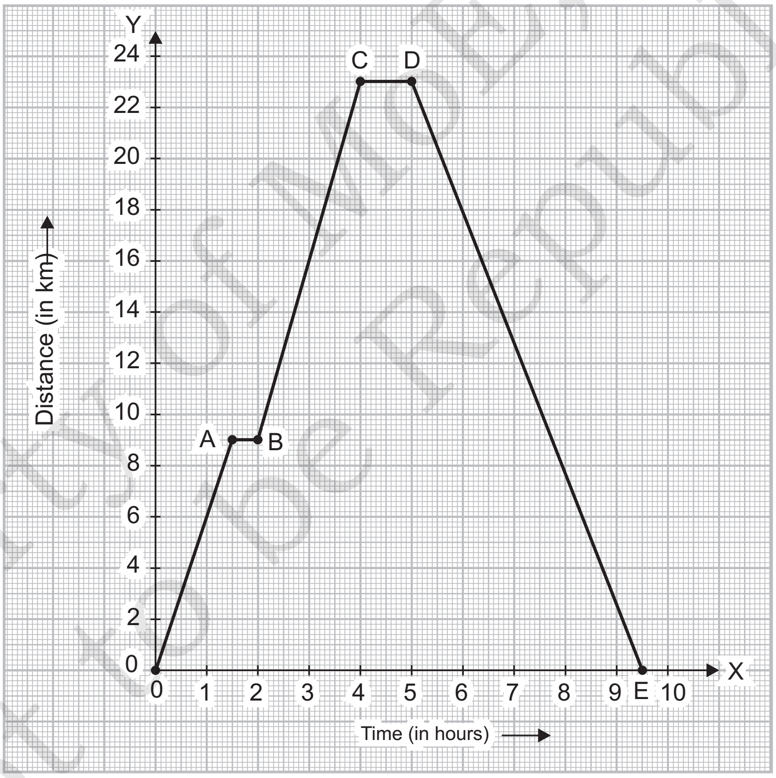
For a journey costing L$ 15, we can subtract the flat rate of L$ 2 to see that L$ 13 has been added on for distance travelled.

As each L$ 1 accounts for 2 miles: 13 × 2 = 26 miles.

EXERCISE 11.6

1. What is the difference between rate and ratio?
2. Ella earns L$ 30000 working 15 hours. Find the unit rate.
3. A man travels 400 kilometers in 10 hours, find the unit rate.
4. Henry can type 200 words in 8 minutes. Working at the same rate, how long does he take to type 800 words?
5. 15 boys can do a piece of work in 60 days. Working at the same rate, how many boys will be required to complete the same work in 20 days?
6. The graph in the figure shows the journey of Albert during a sponsored walk. Describe the parts of the journey shown by:

   (*a*) OA (*b*) AB (*c*) BC (*d*) CD (*e*) DE



7. The graph in the figure shows the journey made by two people, Lisa and Daniel.

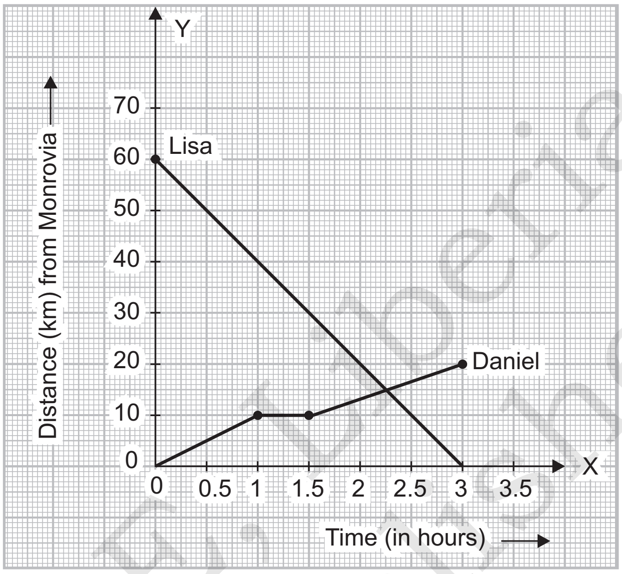
   Daniel sets out from Monrovia at mid-day to travel to Caldwell. At the same time, Lisa sets off from Caldwell and travels to Monrovia at a constant speed. From the graph answer the following questions.

(*i*) What is the distance between Monrovia and Caldwell?

(*ii*) Which traveller is travelling by car and which by cycle? How did you decide?

(*iii*) At what time and where do they meet?

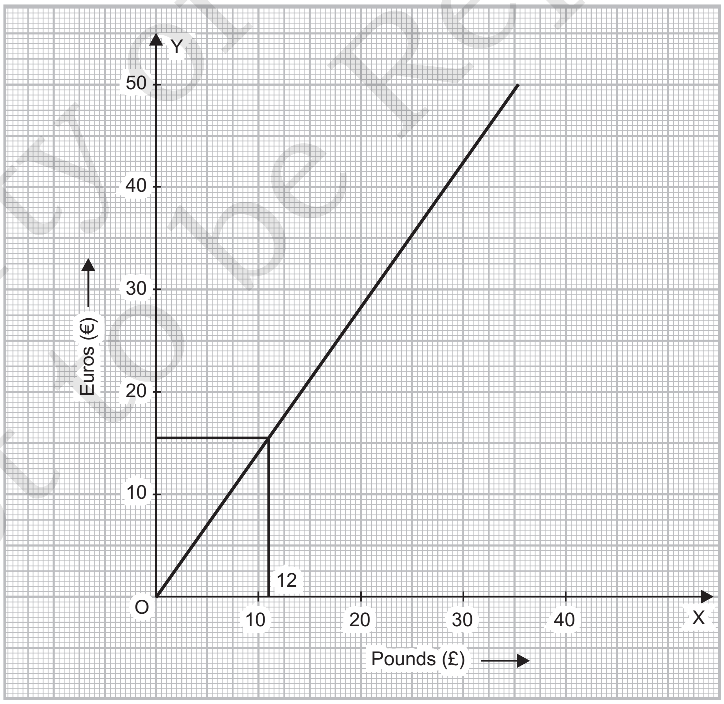
(*iv*) What is Lisa's average speed on the journey?



8. The conversion graph can be used to change between pound (£) and Euros (€).

(*a*) Use the graph to change 30 Pounds to Euros.

(*b*) Use the graph to change 16 Euros to Pounds.

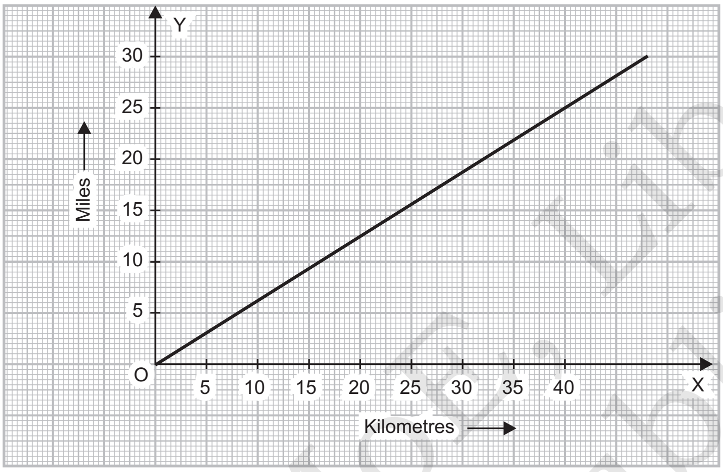


9. A conversion graph for kilometers and miles is shown.

(*a*) Use the graph to convert 40 kilometres to miles.

(*b*) Use the graph to convert 10 miles to kilometres.

(*c*) Convert 200 kilometres to miles.



10. Use the following information to sketch a conversion graph

°C	0	60	100
°F	0	140	212

(*a*) Use graph to change 100°F to °C.

(*b*) Use graph to change 76°C to °F.

(*c*) Convert 200°C to °F.

C. PERCENTAGES

11.13 PERCENTAGE

The word *'percentage'* means 'per hundred' or 'out of hundred'. It is denoted by the symbol % (read as 'percent').

For example: 37 percent = 37% = 37 out of 100 = $\frac{37}{100}$

- **Converting a percentage into a fraction**

  Drop the percentage sign (%) and divide by 100

  For example: $53\% = \frac{53}{100}$

- **Converting a fraction into a percentage**

  Multiply the fraction by 100 and put the percentage sign %.

  For example:

$$\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$$

$$\frac{4}{5} = \left(\frac{4}{5} \times 100\right)\% = 80\%$$

- **Converting a ratio into a percentage**

  First convert the ratio into a fraction and then convert the fraction into a percentage.

  For example:

$$a : b = \frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$$

$$5 : 8 = \frac{5}{8} = \left(\frac{5}{8} \times 100\right)\% = \frac{125}{2}\% = 62.5\%$$

- **Percentage of a quantity**

$$r\% \text{ of } x = \frac{r}{100} \times x$$

  For example:

$$16\% \text{ of } 25 \text{ litres} = \frac{16}{100} \times 25 \text{ litres} = 4 \text{ litres}$$

- **One quantity as a percentage of another quantity**

  To compare two quantities a and b (in same units), we express one as a percentage of the other. Thus,

$$a = \frac{a}{b} \text{ of } b = \left(\frac{a}{b} \times 100\right)\% \text{ of } b$$

$$b = \frac{b}{a} \text{ of } a = \left(\frac{b}{a} \times 100\right)\% \text{ of } a$$

- **Percentage increase/decrease in a quantity**

$$\text{Percentage increase} = \left(\frac{\text{Increase in quantity}}{\text{Original quantity}} \times 100\right)\%$$

$$\text{Percentage decrease} = \left(\frac{\text{Decrease in quantity}}{\text{Original quantity}} \times 100\right)\%$$

- **Increase by x%**

New (increased) quantity

$$= \text{Original quantity} + \text{Increase in quantity}$$
$$= \text{Original quantity} + x\% \text{ of original quantity}$$
$$= \text{Original quantity} + \frac{x}{100} \text{ of original quantity}$$
$$= \left(1 + \frac{x}{100}\right) \text{ of original quantity}$$

Note: The above result is useful in problems on growth.

- **Decrease by x%**

New (decreased) quantity

$$= \text{Original quantity} - \text{Decrease in quantity}$$
$$= \text{Original quantity} - x\% \text{ of original quantity}$$
$$= \text{Original quantity} - \frac{x}{100} \text{ of original quantity}$$
$$= \left(1 - \frac{x}{100}\right) \text{ of original quantity}$$

Note: The above result is useful in problems on decay, depreciation etc.

Percentage error: Where there is an approximation, there is an error. The percentage error in measuring two numbers is defined as % error

$$= \frac{\text{Difference between two numbers}}{\text{Original number}} \times 100$$
$$= \frac{\text{Error}}{\text{Original number}} \times 100$$

where error = difference between two numbers.

Example 24. *Express 18 hours as a percentage of 3 days.*

Solution. Since 1 day = 24 hours, we have 3 days = 3 × 24 = 72 hours

We have to express 18 hours as a percentage of 72 hours.

Required percentage $= \left(\frac{18}{72} \times 100\right)\% = 25\%$

Example 25. *The size of a bag that could hold 5 kg of sugar has now been increased so that it can hold 6 kg. What is the percentage increase in size?*

Solution. Original capacity of bag = 5 kg

New capacity of bag = 6 kg

$\Rightarrow$ Increase in capacity = 6 kg – 5 kg = 1 kg

Percentage increase in capacity

$$= \left(\frac{\text{Increase in capacity}}{\text{Original capacity}} \times 100\right)\%$$

$$= \left(\frac{1}{5} \times 100\right)\% = 20\%.$$

Example 26. *A number is increased by 10% and then it is decreased by 10%. Find the net increase or decrease percentage.*

Solution. Let the original number be 100. It is increased by 10%. Therefore,

$$\text{Increased number} = \left(1 + \frac{10}{100}\right) \times 100 = \frac{11}{10} \times 100 = 110$$

This number is decreased by 10%

$$\therefore \quad \text{Decreased number} = \left(1 - \frac{10}{100}\right) \times 110 = \frac{9}{10} \times 110 = 99$$

which is less than the original number.

Net decrease = 100 – 99 = 1

Therefore, net percentage decrease

$$= \left(\frac{\text{Net decrease}}{\text{Original number}} \times 100\right)\%$$

$$= \left(\frac{1}{100} \times 100\right)\% = 1\%$$

EXERCISE 11.7

1. A man spends 92% of his monthly income. If he saves L$ 220 per month, what is his monthly income?
2. The value of a machine depreciates every year by 10%. What will be its value after 2 years if its present value is L$ 50,000?
3. Samuel used the value of p = 3.14 in a certain calculation whereas the actual value of p is 3.141592654. Find the percentage error up to 2 decimal places.

REVIEW EXERCISE

1. A Junior Football Club in Monrovia has thirty members. Their ages are given below. Make a frequency distribution table for it.

13, 17, 13, 13, 14, 15, 15, 14, 16, 14, 16, 17, 16, 16, 13, 15, 16, 15, 15, 14, 14, 15, 15, 13, 13, 15, 14, 13, 16, 17.

2. Represent the following data in form of a histogram.

Class interval	10–20	20–30	30–40	40–50	50–60	60–70
Frequency	1	5	4	8	5	2

3. The scores in mathematics test (out of 25) of 15 pupils is as follows:

19, 25, 23, 20, 9, 20, 15, 10, 5, 16, 25, 20, 24, 12, 20.

Find the mode and median of this data. Are they same?

4. The heights of 5 pupils in a group are:

152 cm, 170 cm, 156 cm, 164 cm and 158 cm

(*a*) Find the mean height

(*b*) How many pupils have heights more than the mean height?

5. Find the mode of the following data:

20, 21, 25, 22, 17, 22, 13, 15, 23, 21, 9, 10, 22, 20, 30.

6. Find the median of the following data:

2, 5, 3, 2, 4, 5, 2, 4, 6, 8, 7, 9

7. Find the mean temperature of a city/village for the last month.

8. Find the mean of 6.5, 8.2, 9.4, 4.6, 7.8 and 4.9.

9. The heights (in cm) of 20 pupils is given below:

106, 110, 123, 125, 117, 120, 112, 115, 110, 120, 115, 102, 115, 115, 109, 107, 115, 101, 108, 129.

Represent the above data using the stem and leaf plot.

10. The number of pupils passed in Grade-10 (Semester-II) in two consecutive years is as under.

Standard	V	VI	VII	VIII	IX	X
No. of pupils passed in 2020	90	100	115	115	110	100
No. of pupils passed in 2021	80	95	100	105	110	95

Draw a double bar graph using the above data.

11. The number of pupils who opted for coaching classes in various activities organised by the school were as follow:

Activity	Cricket	Photography	Karate	Music	Dance
No. of pupils	75	20	30	45	10

Represent this data on a pie chart.

12. If the age of 9 pupils in a team is 12, 13, 11, 12, 13, 12, 11, 12, 12. Then find the average age of pupils in the team.

13. Find the average of first four multiples of 2.

14. Find the average of 6, 13, 17, 21, 23.

15. Two numbers are in the ratio of 3 : 4. When 8 is subtracted from each, the ratio becomes 2 : 3. Find the numbers.

16. A uniform rod of length 8 m has mass 20 kg. What is the mass per metre?

17. A tap leaks at a rate of 2 cm^3 per second. How long will it take to fill a container of 45 litres capacity (1 litre = 1000 cm^3).

18. Juliet measured the length of her classroom and obtained 4.99 m with a percentage error of 5%. Her own measurement was smaller than the original length. What was the actual length of the room?

MULTIPLE CHOICE QUESTIONS (MCQs)

1. The mean of the numbers 4, 3, 3, x is 5. Find x.

(a) 20 (b) 10 (c) 5 (d) 4

2. The ages in years of eight boys are 14, 14.5, 12, 11.5, 15, 13, 10.5, 13.5. What is their average age?

(a) 14 (b) 13 (c) 12 (d) 11

Directions: *The marks obtained by 10 boys in a test are 0, 1, 3, 3, 5, 7, 8, 9, 9, 9. Use this information to answer questions 3 to 4.*

3. Find the median score.

(a) 3 (b) 5 (c) 6 (d) 7

4. Calculate the mean score.

(a) 4.4 (b) 5.4 (c) 6 (d) 6.4

5. The marks obtained by six boys in a test are 20, 25, 15, 30, 28 and 16. Find the mean mark.

(a) 19.33 (b) 22.33 (c) 23.20 (d) 26.40

6. The ratio 9 : x is equivalent to 36 : 20. What is the value of x?

(a) 4 (b) 5 (c) 6 (d) 8

7. The angles of a triangle are in the ratio 3 : 2 : 1. Find the value of the smallest angle.

(a) 30° (b) 45° (c) 60° (d) 90°

8. A map of a large town is drawn to the sacle of 1 : 100,000. What is the distance in kilometres (km) represented by a line segment 4 cm long on the map?

(a) 0.04 km (b) 0.4 km (c) 4 km (d) .40 km

9. Ten pupils in Monrovia High School took 9 days to weed the school compound. How long would 15 pupils take to weed the compound if they worked at the same rate?

(a) 5 days (b) 6 days (c) $13\frac{1}{2}$ days (d) 14 days

10. The scale of a map is 1 : 100,000. What is the distance (in kilometres) between two towns 4 cm apart on the map?

(a) 0.04 (b) 0.4 (c) 4.0 (d) 400

11. A tank contains 250 litres of water. If 96 litres is used, what percentage of the original quantity is left?

(a) 61.6% (b) 60.5% (c) 59.0% (d) 54.2%

12. Ella obtained 150 marks out of 240 marks in an English test. What was her percentage score?

(a) 33.33% (b) 36.5% (c) 41.67% (d) 62.5%

RECAP AT A GLANCE

- Statistics is the branch of Mathematics which deals with the collection, presentation and analysis of numerical data
- The mode of a set of observations is the observation that occurs most often.
- Median refers to the value which lies in the middle of the data.
- A plot where each data value is split into a "leaf" and a "stem" is called a stem and leaf plot.
- The average of a list of data is the expression of the central value of a set of data.
- A ratio is a comparison of two or more similar quantities.
- A rate is a ratio that is used for comparing two different kinds of quantities which have different units.
- Conversion graphs are straight line graphs that show a relationship between two units.
- The word *'percentage'* means 'per hundred' or 'out of hundred'. It is denoted by the symbol % (read as 'percent').

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